GOVERNMENT OF ANDHRA PRADESH

COMMISSIONERATE OF COLLEGIATE EDUCATION

SOLID GEOMETY PLANES

PLANE PASSING THOUGH THREEPOINTS

MATHEMATICS

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THEOREM : The vectors equations of the plane passing through the points A,B,C having position vectors a,b,c is [r-a,b-a,c-a]=0

Proof: IF P is a point in the plane passing through the points A,B,C then $\overrightarrow{AP}, \overrightarrow{AB}, \overrightarrow{AC}$ are coplanar and hence $[\overrightarrow{AP}, \overrightarrow{AB}, \overrightarrow{AC}]=0$

Conversely suppose that if P is a point such that $[\overrightarrow{AP}, \overrightarrow{AB}, \overrightarrow{AC}]=0$

then $\overrightarrow{AP}, \overrightarrow{AB}, \overrightarrow{AC}$ are coplanar and hence P lies in the plane passing through the points A,B,C.

Thus the locus of P is $[\overrightarrow{AP}, \overrightarrow{AB}, \overrightarrow{AC}]$=0

∴ The vector equation of the plane is

$$[\overrightarrow{AP}, \overrightarrow{AB}, \overrightarrow{AC}] = 0$$

If $\overrightarrow{OP}$=r, then the vector equation of the plane is $[\overrightarrow{AP}, \overrightarrow{AB}, \overrightarrow{AC}] = 0$

$\Rightarrow$ $[\overrightarrow{OP} - \overrightarrow{OA}, \overrightarrow{OB} - \overrightarrow{OA}, \overrightarrow{OC} - \overrightarrow{OA},]$=0

$\Rightarrow$ [r-a,b-a,c-a]=0

THEOREM: The equation of the plane passing through $(x_1, y_1, z_1),(x_2,y_2,z_2),(x_3,y_3,z_3)$ is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$$

Proof: Let $a=x_1i+y_1j+z_1k$ $\quad$ $b=x_2i+y_2j+z_2k$

$C=x_3i+y_3j+z_3k$ and $r=xi + yj + zk$

Now $r-a=(x-x_1)i+(y-y_1)j+(z-z_1)k$,

$b-a=(x_2-x_1)i+(y_2-y_1)j+(z_2-z_1)k$,

$c-a=(x_3-x_1)i+(y_3-y_1)j+(z_3-z_1)k$

Equation of the plane is $[\, r-a, b-a, c-a] = 0$

$$\Rightarrow \begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

Second proof:

Equation of a plane passing through

(x_1, y_1, z_1) is $a(x-x_1)+b(y-y_1)+c(z-z_1)=0 \rightarrow$ (1)

If plane (1), passing through (x_2,y_2,z_2), then

$a(x_2-x_1)+b(y_2-y_1)+c(z_2-z_1)=0 \rightarrow$ (2)

If plane (1), passing through (x_3,y_3,z_3), then

$a(x_3-x_1)+b(y_3-y_1)+c(z_3-z_1)=0 \rightarrow$ (3)

Eliminating a,b,c from (1),(2),(3), we get

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0 \rightarrow (4)$$

Equation (4) is a first degree equation in x, y, z and satisfied by the given points.

Therefore it represents a plane passing through the given points.

EX: Find the equation of the plane passing through the points (2,2,-1),(3,4,2),(7,0,6)

Formula: The equation of the plane passing through $(x_1, y_1, z_1),(x_2,y_2,z_2),(x_3,y_3,z_3)$is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

Given points are $(x_1, y_1, z_1)= (2,2,-1)$

$(x_2,y_2,z_2)= (3,4,2)$

$(x_3,y_3,z_3)= (7,0,6)$

$\therefore$ Equation of the plane passing through given points is $\begin{vmatrix} x-2 & y-2 & z+1 \\ 3-2 & 4-2 & 2+1 \\ 7-2 & 0-2 & 6+1 \end{vmatrix}$=0

$$\begin{vmatrix} x-2 & y-2 & z+1 \\ 1 & 2 & 3 \\ 5 & -2 & 7 \end{vmatrix}=0$$

$\Rightarrow$(x- 2)(14+6)-(y-2)(7-15)+(z+1)(-2-10)=0

$\Rightarrow$ (x- 2)(20)-(y-2)(-8)+(z+1)(-12)=0

$\Rightarrow$20x-40+8y-16-12z-12=0

$\Rightarrow$20x+8y-12z-68=0

$$\Rightarrow \quad (x-2)(20)-(y-2)(-8)+(z+1)(-12)=0$$

$$\Rightarrow 20x-40+8y-16-12z-12=0$$

$$\Rightarrow 20x+8y-12z-68=0$$

$$\Rightarrow 5x+2y-3z-17=0$$

$\therefore$ Equation of the required plane is

$$5x+2y-3z-17=0$$

$\therefore$

Second method

EX: Find the equation of the plane passing through the points (2,2,-1),(3,4,2),(7,0,6)

Sol:

The equation of the plane through the points (2,2,-1) is $a(x-2)+b(y-2)+c(z+1)=0$

(3,4,2) lies in the plane

(3,4,2) lies in the plane

$\Rightarrow a(3-2)+b(4-2)+c(2+1)=0$

$\Rightarrow a(1)+b(2)+c(3)=0$

$\Rightarrow a+2b+3c=0 \longrightarrow (1)$

(7,0,6) lies in the plane

$\Rightarrow a(7-2)+b(0-2)+c(6+1)=0$

$\Rightarrow a(5)+b(-2)+c(7)=0$

From (1) and (2)

	a		b	c	
2		3		1	2
-2		7		5	-2

$$\Rightarrow \frac{a}{14+5} = \frac{b}{15-7} = \frac{c}{-2-10}$$

$$\Rightarrow \frac{a}{20} = \frac{b}{8} = \frac{c}{-12} \Rightarrow \frac{a}{5} = \frac{b}{2} = \frac{c}{-3}$$

$\therefore$The equation to the required plane is

$\Rightarrow$ 5(x-2)+2(y-2)-3(z+1)=0

$\Rightarrow$ 5x+2y-3z-17=0